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The Effect of a Splitter Plate on the Symmetry of Separated Flow Around a Delta Wing of Low Aspect Ratio

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THE EFFECT OF A SPLITTER PLATE ON THE SYMMETRY OF SEPARATED FLOW AROUND A DELTA WING OF LOW ASPECT RATIO

[VLIYANIE RAZDELITEL'NOI PLASTINY NA SIMMETRICHNOST' OTRYVNOGO OBTEKANIYA TREUGOL'NOGO KRYLA MALOGO UDLINENIYA]

by

S. B. Zakharov

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AUTHOR'S SUMMARY

Using the approximation of the theory of slender bodies an investigation is made into the non-uniqueness of solutions for separated flow around a delta wing of low aspect ratio in the presence of a delta splitter plate on the upper surface of the wing and in the plane of symmetry. An improved method of formulation offers increased accuracy, when applied to the solution of similarity problems in vortex sheets. Some substantially asymmetrical solutions to the problem of separated flow around the wing under symmetrical conditions of flow are obtained, as well as some symmetrical ones. Considerable attention is paid to an explanation of the nature of the bifurcations in solutions and to a qualitative description of hysteresis in the aerodynamic characteristics as a result of a quasi-stationary change in angle of attack and yaw angle by the use of a simplified mathematical model of a vortex sheet 'vortex and cut'. Great Britain (JID)

Russian Translations

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Following the results of experimental investigations, a symmetrical separated flow picture around a model of a delta wing of low aspect ratio in the absence of yaw angle is observed only over a range of angles of attack α , less than a certain critical value α_* . For values of the angle of attack $\alpha > \alpha_*$ the separated flow picture is significantly asymmetrical, and significant lateral forces and moments act on the model of the wing under investigation. As may be expected, a loss of stability of the symmetrical arrangement of the vortex sheets arising from the sharp lateral edges of the wing is responsible for the observed phenomena. These develop and interact strongly with themselves and the surface of the model. At small angles of attack the vortex sheets are located close to the edges of the wing and for practical purposes do not interact with each other, and their symmetrical arrangement is stable. The effect being considered is observed in models of wings having an aspect ratio $\lambda < 1$. In models of delta wings having $\lambda > 1$, breakdown of the vortices takes place[1] with increase in angle of attack before the beginning of asymmetry in the separated flow picture.

An experimental investigation of the phenomenon under consideration is attended by considerable technical difficulties associated with the need to prepare sufficiently rigid models of plane delta wings of low and ultra-low aspect ratio. At first glance, the simplest way of providing the required rigidity in a plane model of a delta wing without disturbing the separated flow picture, and preserving the inherent aerodynamic characteristics of a plane delta wing in the absence of yaw angle consists of placing a thin rib (a delta shaped splitter plate) with either a sharpened or rounded edge in the plane of symmetry, on the upper or lower side of the model of the wing[2].

The aim of the present work is to investigate numerically the problem of separated flow around a plane delta wing with a delta splitter plate, in those cases when the latter is placed in the plane of symmetry on the upper side of the wing; we will assume that it has a rounded edge. This assumption is made in order to simplify the problem, and allows us, as a first approximation, to ignore the separation of the flow from the splitter plate, which necessarily takes place at a sharp edge in the event of break-up of the symmetrical separated flow around the wing.

1 FORMULATING THE PROBLEM

Since the phenomenon under consideration - the occurrence of significant asymmetry in the separated flow - is observed in experiments on models of wings having an aspect ratio $\lambda < 1$, there is complete justification for the use of the approximations of the theory of slender bodies, and in accordance with these

approximations, the use of numerical methods for the calculation of separated flow around wings, assuming a global inviscid nature of the flow. The formulation of the problem within the framework of the theory of slender bodies has been adequately described in many references, including Refs 3 and 4, and there is no need for a detailed account of it. Let us make note only of some basic assumptions: as a consequence of the low aspect ratio the three-dimensional steady problem reduces to a two-dimensional unsteady problem of separated flow in a stream of incompressible fluid of broadening contour in the planes of the cross-section of the wing (the hypothesis of plane sections). In the case of a wing of conical geometry, the three-dimensional separated flow is taken to be conical, and the corresponding two-dimensional problem to be of similarity type; the flow is considered to be inviscid, and the separation of the flow with the formation of spiral vortex sheets takes place only at the sharp lateral edges of the wing. The latter solution of the problem, as already noted above, includes a simplifying assumption.

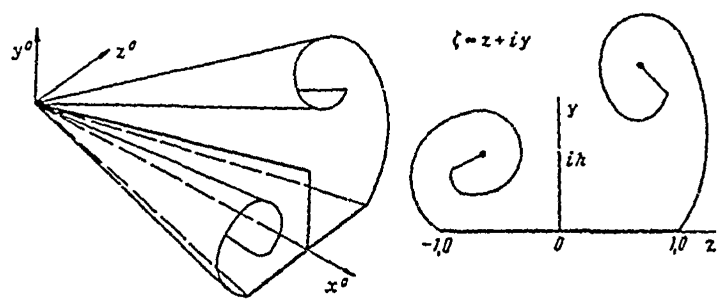

Fig 1

Let $Ox^oy^oz^o$ be a system of orthogonal Cartesian co-ordinates associated with the wing, having the origin at its apex (Fig 1). The x^o axis is directed along the central chord of the wing, the y^o axis is perpendicular to the plane of the wing, and the z^o axis is parallel to the span. In the problem under consideration, *ie* separated flow around the combination of a delta wing and a splitter plate, the defining parameters are: the relative height of the splitter plate $h = \theta/\chi$, the relative angle of attack $\alpha = \alpha^o/\chi$ and yaw angle $\beta = \beta^o/\chi$. Here χ is the half-angle at the apex of the wing, θ is the angle at the apex of the splitter plate, α^o is the angle of attack, and β^o is the yaw angle. When $\beta = 0$, giving symmetrical solutions, the parameter h drops out of the set of defining parameters; the solutions obtained in this case, depending only on α, are those of the problem of separated flow around a plane delta wing having zero yaw angle.

Let us consider the flow picture in a given plane of cross-section of the wing (Fig 1). Let us take the complex similarity variable $\zeta = z + iy$, where $z = z^o/\ell$, $y = y^o/\ell$, and ℓ is the half-span of the wing at the given section.

In the following, in order to perform what is necessary for the numerical solution of the problem of complex potential and complex flow velocity, let us use the conformal mapping of the surface of the given contour onto the surface of a unit circle in the auxiliary plane of the complex variable $\sigma = \xi + i\eta$:

$$\sigma(\zeta) = \frac{1}{a}\left\{\sqrt{\zeta^2 - 1} + \sqrt{(\sqrt{\zeta^2 - 1} - ib)^2 + a^2} - ib\right\} .$$

$$a = \frac{\sqrt{1 + h^2} + 1}{2}, \qquad b = \frac{\sqrt{1 + h^2} - 1}{2} .$$

With such a mapping the point $\sigma = i$ corresponds to the point $\zeta = ih$ (the edge of the splitter plate), and correspondingly, the points σ_1^o, and σ_2^o, to the points $\zeta_1^o = 1$ and $\zeta^o_2 = -1$ (the lateral sharp edges of the wing).

2 METHOD OF SOLUTION

For an effective numerical solution of the problem under consideration, which belongs to a class of problems in which non-unique solutions for separated flow are possible, a method or combination of methods is required which will allow not only the achievement of solutions, but also an investigation into them for stability against small disturbances. These requirements are met by the method of establishment[5], the basis of which is to obtain a numerical solution of Cauchy's problem for the nonlinear integro-differential equation of the evolution of a tangential discontinuity in velocity[6].

The numerical algorithm of the method of establishment[5] comprises integration of the equations of motion of discrete vortices approximating to a continuous vortex sheet, according to Euler's method (formulae of numerical integration of the first order of accuracy)

$$\zeta^n(t + \Delta t) = \zeta^n(t) + \bar{V}^n \Delta t \qquad (1)$$

as well as successive smoothing along the co-ordinates with each time interval, using the regularising operator

$$\zeta^n = \tfrac{1}{4}(\zeta^{n-1} + 2\zeta^n + \zeta^{n+1}) . \qquad (2)$$

Here t and Δt are the rime and the time interval, ζ^n are the complex co-ordinates of the discrete vortices in the ζ plane, V^n is the complex velocity of the stream at the location points of the vortices, the stroke over the variable indicating the complex conjugate.

In the present work an improved numerical algorithm was used, in which the operator (2) was replaced by the operator

$$\zeta^n = \frac{1}{16}\left(\zeta^{n-2} + 4\zeta^{n-1} + 6\zeta^n + 4\zeta^{n+1} + \zeta^{n+2}\right) , \tag{3}$$

which is equivalent in its effect to the repeated application at each step of the operator (2).

The operator (3), besides using a smoothing function, serves the purpose of a correcting operator for formula (1), in fact increasing by an order the accuracy of the method of numerical integration for the solution of similarity problems by the method of establishment, with practically no increase in the computation time. This latter property of the operator (3) is associated with the existence of completely defined relationships between the integration step of time Δt and the discretisation step of the vortex sheet $\Delta\zeta$

$$\Delta\zeta^{n+1,n} = \zeta^{n+1} - \zeta^n = \zeta^n(t + \Delta t) - \zeta^n(t) = \bar{V}^n\Delta t .$$

In other words, at each integration step the discrete vortices continue to move to points in which neighbouring discrete vortices are situated. The vortices move along the vortex sheet, whose configuration no longer changes.

The well-recommended mathematical model 'vortex and cut' of the core of a spiral vortex sheet[3] was employed to approximate to the internal rotation of a vortex sheet. The coordinates and the circulations of the discrete vortices arising from the sharp lateral edges of the wing were calculated, taking into account the asymptote of the vortex sheet adjacent to the separation points.

3 RESULTS OF THE CALCULATION

In spite of the limited nature of the calculations carried out, some definite conclusions may be reached. With $\beta = 0$ and $\alpha < \alpha_*(h)$ there exists a unique symmetrical solution, as first obtained in Ref 3, stable against arbitrary small disturbances. With $\beta = 0$ and $\alpha > \alpha_*(h)$, there exist at the same time as this symmetrical solution two asymmetric solutions as mirror images in the y axis; these are stable against arbitrary small disturbances. While a symmetrical solution is always stable against small symmetrical disturbances, there

may be instability to arbitrary small (asymmetrical) disturbances. The latter depends upon the values of two quantities: the relative height of the splitter plate h and $\alpha - \alpha_*(h)$. The asymmetry increases with the angle of attack.

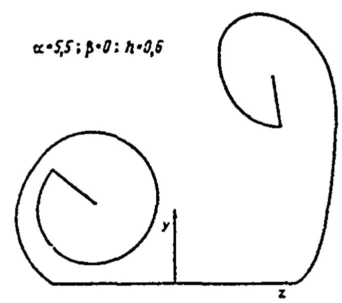

Fig 2

As an example, Fig 2 shows the configuration of the vortex sheets corresponding to one of the asymmetrical solutions for the following values of the defining parameters: $h = 0.6$, $\alpha = 5.5$, $\beta = 0$.

With $\beta = 0$ and $h = 0$, it was not possible to obtain asymmetrical solutions. An unsuccessful analogous experiment to obtain numerically an asymmetrical flow picture around an isolated plane delta wing of aspect ratio $\lambda = 0.25$ is described in Ref 7; a three-dimensional panelling method was employed for the calculation of the flow.

4 BIFURCATION ANALYSIS

The calculations described previously using a complicated mathematical model of a spiral vortex sheet are quite time-consuming and require a significant expenditure of machine time. On the other hand, the simple model[8] of a spiral vortex sheet, 'vortex and cut' is well-known and is applied widely in parametric calculations; it is regularly used qualitatively, and under individual conditions quantitatively, to describe the separated flow picture around wings of small aspect ratio.

Within the framework of this model the whole vortex sheet is changed into a single discrete vortex, joined to the sharp edge of the wing by a mathematical ('feeding') cut. Let ζ_1, ζ_2 and Γ_1, Γ_2 be the complex coordinates and the circulations of the discrete vortices, right and left respectively, while σ_1, σ_2 are the corresponding complex coordinates of the vortices in the auxiliary plane

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of the complex variable σ . The complex flow potential then takes the form

$$\omega(\sigma) = -i\frac{a\alpha}{2}\left(\sigma - \frac{1}{\sigma}\right) + \frac{a\beta}{2}\left(\sigma + \frac{1}{\sigma}\right) + \frac{\Gamma_1}{2\pi i}\ln\frac{\sigma - \sigma_1}{\sigma - \frac{1}{\bar{\sigma}_1}} + \frac{\Gamma_2}{2\pi i}\ln\frac{\sigma - \sigma_2}{\sigma - \frac{1}{\bar{\sigma}_2}} .$$

In order to determine the four unknowns, *ie* the two real, Γ_1, Γ_2 and the two complex, ζ_1, ζ_2 we have the system of algebraic equations

$$\left.\frac{dw}{d\sigma}\right|_{\sigma=\sigma_1^o} = 0 , \qquad \left.\frac{dw}{d\sigma}\right|_{\sigma=\sigma_2^o} = 0 , \tag{4}$$

expressing the Chaplygin-Joukowski postulate concerning the finiteness of the velocity at the sharp lateral edges of the wing, and the two equations

$$\begin{aligned} \bar{\zeta}_1 &= \tfrac{1}{2} + \tfrac{1}{2}\lim_{\zeta\to\zeta_1}\left\{\frac{dw}{d\sigma}\frac{d\sigma}{d\zeta} - \frac{\Gamma_1}{2\pi i}\frac{1}{\zeta - \zeta_1}\right\} , \\ \bar{\zeta}_2 &= -\tfrac{1}{2} + \tfrac{1}{2}\lim_{\zeta\to\zeta_2}\left\{\frac{dw}{d\sigma}\frac{d\sigma}{d\zeta} - \frac{\Gamma_2}{2\pi i}\frac{1}{\zeta - \zeta_2}\right\} , \end{aligned} \tag{5}$$

which express the conditions for the absence of a total force acting on the 'vortex and cut' system.

To solve the systems of (4) to (5), a simple method of iteration (relaxation) was used, to allow solutions to be obtained which are stable when subjected to a defined class of small disturbances, and to enable one to judge the stability of these solutions against a wider class of small disturbances. The latter is necessary for an investigation into the stability against small asymmetrical disturbances of existing symmetrical solutions of the problem under consideration, when $\beta = 0$. There is a shortcoming in the method employed, that is, the impossibility of obtaining unstable solutions for both asymmetrical and symmetrical small disturbances, although the latter, as absolutely unstable solutions, are not of any great interest in practical applications.

The method is as follows: at the start of iteration the coordinates of the vortices $\zeta_1^{(o)}$, $\zeta_2^{(o)}$ are set up quite arbitrarily. The circulations of the vortices $\Gamma_1^{(o)}$, $\Gamma_2^{(o)}$ are determined from the equations (4). The right-hand sides of the equations (5) $f_1^{(0)}(\zeta_1^{(0)},\zeta_2^{(0)},\Gamma_1^{(0)},\Gamma_2^{(0)})$, $f_2^{(0)}(\zeta_1^{(0)},\zeta_2^{(0)},\Gamma_1^{(0)},\Gamma_2^{(0)})$ are calculated using the initial coordinates and circulations of the discrete vortices. By way of an initial approximation to the next iteration we take the complex coordinates of the discrete vortices, calculated from the expressions

$$\zeta_1^{(1)} = (1 - \varepsilon_1^{(0)})\zeta_1^{(0)} + \varepsilon_1^{(0)}\bar{\xi}_1^{(0)}, \quad \zeta_2^{(1)} = (1 - \varepsilon_2^{(0)})\zeta_2^{(0)} + \varepsilon_2^{(0)}\bar{\xi}_2^{(0)} ,$$

and here $0 < \varepsilon_1^{(0)}, \varepsilon_2^{(0)} < 1$ are optionally equal real quantities, whose purpose is to speed up the convergence, which will desirably be increased with the number of iterations. The latter is especially important in the region of the bifurcation points of the solution, where, as practice shows, the convergence of the method slows down sharply.

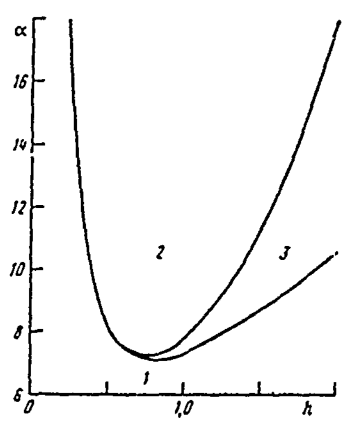

Fig 3

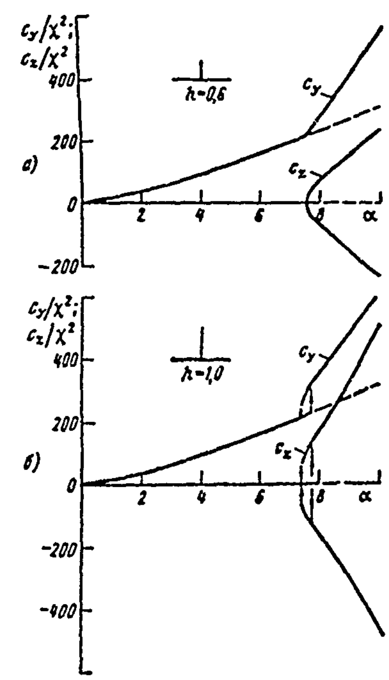

Fig 4

In Fig 3 is shown the bufurcation diagram in the plane of the parameters (h,a) defining the flow, with $\beta = 0$. In region 1 there exists a unique symmetrical solution which is stable against arbitrary small disturbances, as first obtained in Ref 8; it is the solution of the problem of separated flow around a plane isolated delta wing at zero yaw angle, with the 'vortex and cut' model of a vortex sheet. In region 2 there is a symmetrical solution which is unstable against small asymmetrical disturbances, but stable against symmetrical disturbances, allowing the possibility of obtaining it by the method of iteration

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employed, using a programme of symmetrical initial approximation and the condition $\varepsilon_1 = \varepsilon_2$ at each iteration. In region 2, besides a symmetrical solution there exist two asymmetrical solutions which are mirror images in the y axis, and are stable against arbitrary small disturbances. In region 3 there also exist three stable solutions, one symmetrical and two asymmetrical, but in contrast to region 2 all three solutions are stable against arbitrary small disturbances. Apart from these three solutions - as follows from an analysis of the character of a bifurcation point[9] - there must exist in this region two more solutions mirrored in the y axis, which are absolutely unstable and asymmetrical, but which are impossible to obtain by the method employed.

With $\beta = 0$ in the range $0 < h < 0.7$, the bifurcation of the solution takes on a supercritical nature, and in the range $h > 0.7$, is subcritical. The diverse nature of the bifurcations in the ranges of variation shown for the relative height h of the splitter plate leads to a varying degree of dependency of the coefficients of the aerodynamic forces and moments on the angle of attack. In the range $h > 0.7$ with a quasi-stationary change in angle of attack, hysteresis in the aerodynamic characteristics is possible.

As an example, Fig 4 shows the relationships between the normal c_y and lateral c_z forces and the angle of attack α, for two values of relative height $h = 0.6$ (Fig 4a) and 1.0 (Fig 4b). The coefficients of the normal and lateral forces were calculated according to the theorem relating to the momentum according to the formula

$$c_z + ic_y = \chi^2\left\{2\pi i\alpha + 2\pi(a^2 - 1)\beta + ia\Gamma_1\left(\sigma_1 - \frac{1}{\bar{\sigma}_1}\right) + ia\Gamma_2\left(\sigma_2 - \frac{1}{\bar{\sigma}_2}\right)\right\} .$$

Which of the two branches of dependence of $c_z(\alpha)$ is achieved is a matter of chance. A flow picture in which the right-hand vortex is higher than the left corresponds to negative values of c_z.

The existence of two (region 2, Fig 3) or three (region 3) different solutions for $\beta = 0$, which are stable against arbitrary small disturbances, stipulates the existence of the same number of solutions stable against arbitrary small disturbances in a certain symmetrical range $0 < |\beta| < \beta_*(h,a)$ differing from zero yaw angle; this, in its turn, results in a hysteretic relationship between the aerodynamic coefficients and the yaw angle, on applying a quasi-stationary change.

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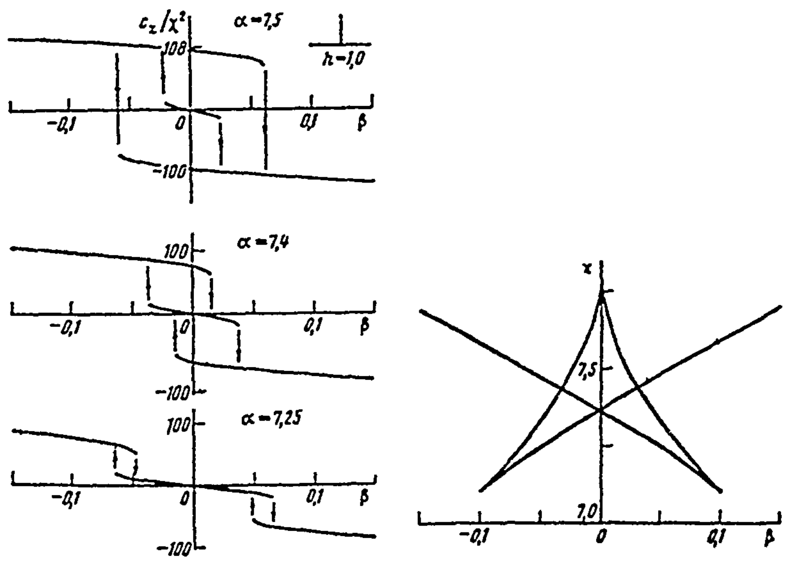

Fig 5

As an example, Fig 5 shows some characteristics of the relationship between the coefficient of lateral force and the yaw angle at $h = 1.0$ for different values of angle of attack. Jumps in the hysteresis loops on applying a quasi-stationary change of yaw angle are possible only in the directions of the arrows shown. Also in Fig 5 is shown the bifurcation diagram corresponding to the chosen value $h = 1.0$, in the plane (β,α), known as 'the butterfly'; this is the projection of the three-dimensional surface $F(\alpha,\beta,c_z) = 0$ onto the plane of the controlling parameters. With $h < 0.7$ the three-dimensional surface $F(\alpha,\beta,c_z) = 0$ represents a variety of 'fold catastrophe', and its projection onto the plane (β,α) of the controlling parameters takes the form of a 'fold'[9].

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